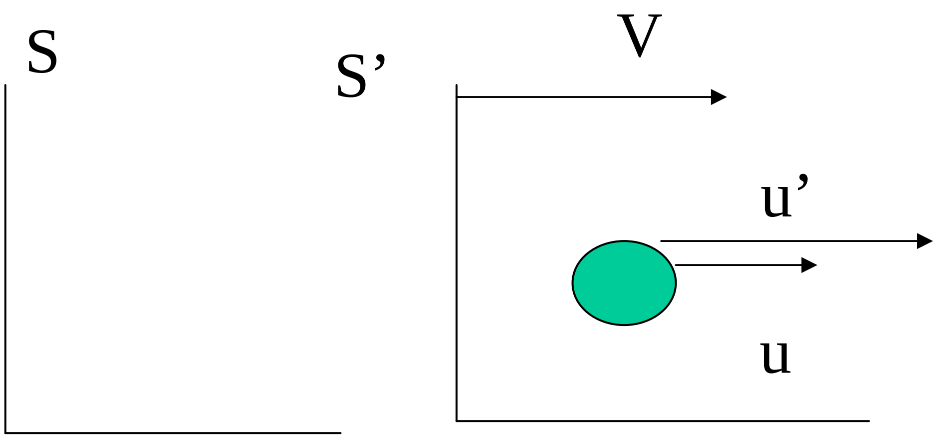
Mass, Energy, and Momentum

- In Relativity, Momentum = mu (u = speed in frame)
- Where $m = \gamma m_o$

Relative velocity

- Why the change in mass?
- Consider the relative velocities between frames:



$$u = \frac{dx}{dt} = \frac{dx}{dt'}\frac{dt'}{dt}$$

$$x = (x' + v't)\gamma$$

$$t' = (t - \frac{vx}{c^2})\gamma$$

Velocity transformation

$$u = \frac{dx}{dt} = \frac{dx}{dt^{'}}\frac{dt'}{dt} =$$

$$\frac{d}{dt^{'}}\{(x'+vt')\gamma\}\frac{d}{dt}\left\{(t+\frac{vx}{c^2})\gamma\right\}$$

$$u = (u'+v)(1+\frac{uv}{c^2})\gamma^2 \quad -> (solve \text{ for u})$$

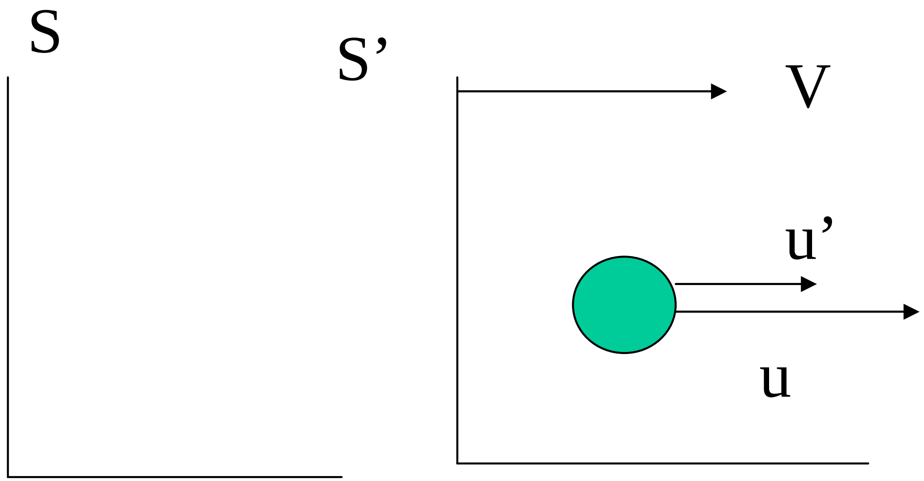
$$u = \frac{(u'+v)}{1+\frac{uv}{c^2}} \text{ or inverse transform :, u --> u', v --> -v}$$

$$\longrightarrow \quad u' = \frac{(u-v)}{1-\frac{u'v}{c^2}}$$

Mass, Energy, and Momentum

If u’ = .9c, and v = .9c,

Using Galilean Relativity, u = 1.8c…not allowed!



$$u = \frac{(u' + v)}{1 + \frac{uv}{c^2}}$$

then u = .994c !

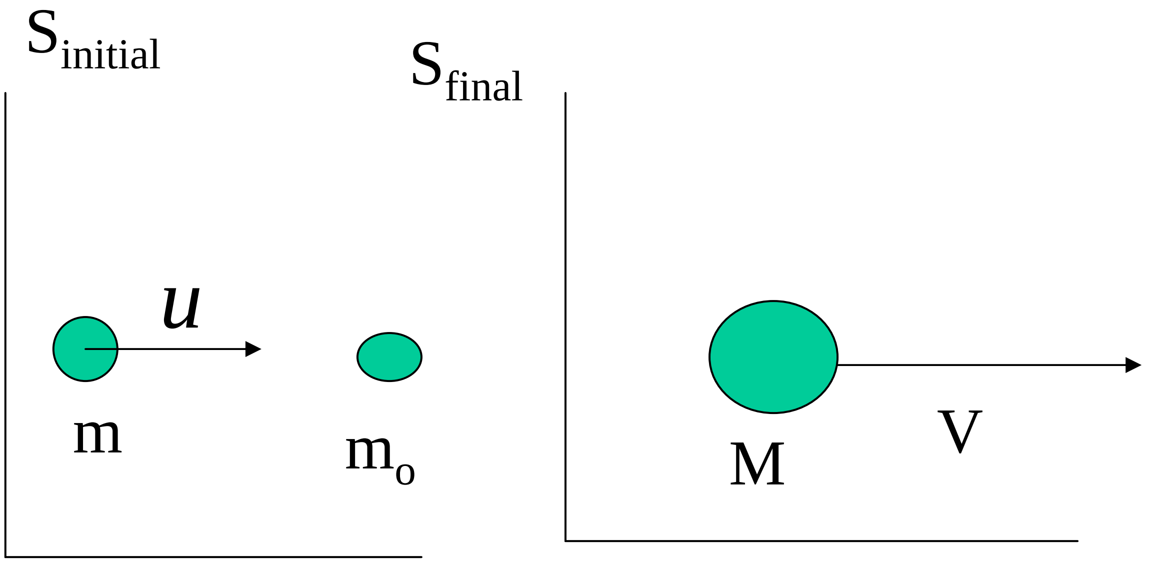
Now Consider a two body collision

- View Collision from Frame S:

$$mu + m_o(0) = Mv$$

$$Also,$$

$$m + m_0 = M$$



Combining gives: $$v = \frac{mu}{m + m_o}$$

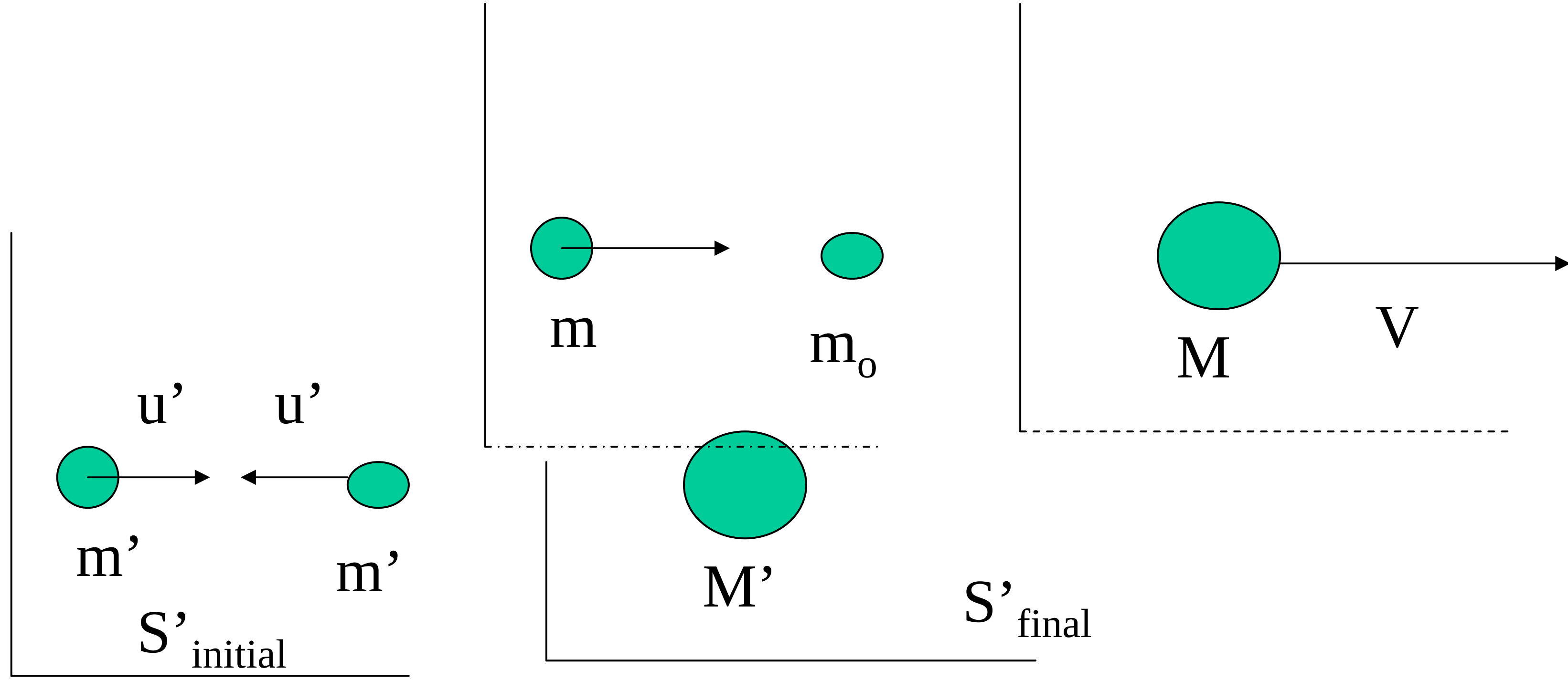
Consider a two body collision

- View Collision from Frame S’ moving with velocity V=u’ relativie to S, so that M is at rest.

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\textit{for}\ \text{mass on right}: u' = \frac{0 - v}{1 - \frac{(0)v}{c^2}} = -v$$

$$\textit{for}\ \text{mass on left: u}' = +\text{v or else M' not at rest!}$$



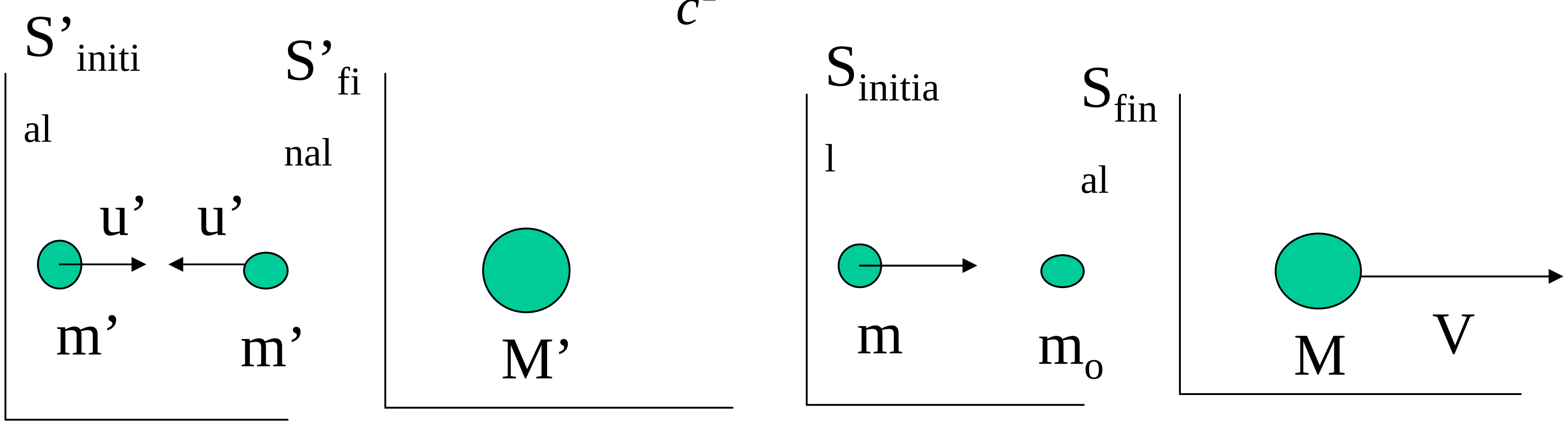
Proof that $m=\gamma m_0$

Now that we have our velocities properly tranformed, lets combine the results of momentum conservation in frame S with the velocity transform equation between S and S':

$$\text{Namely } V = \frac{mu}{m+m_o} \qquad \text{and } u = \frac{u'+v}{1+\frac{u'v}{c^2}} =$$

$$\textit{where } \text{u}' = \text{v}$$

$$\textit{so} \text{ that u} = \frac{2v}{1+\frac{v^2}{c^2}}$$

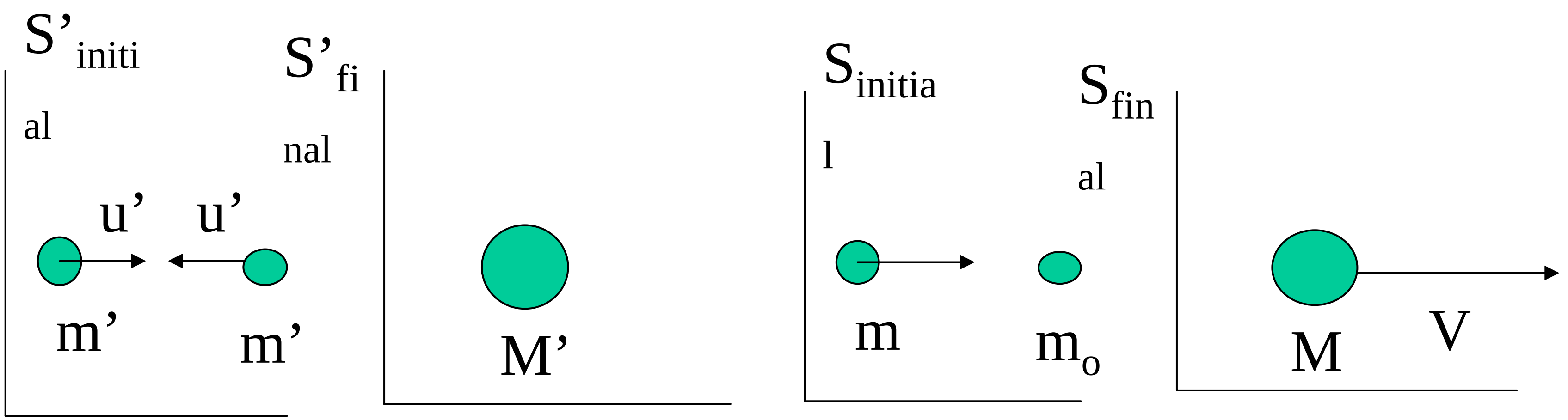


Proof that $m=\gamma m_0$

$$\text{let} \quad v = \frac{mu}{m+m_o}$$

$$u = \frac{2\left(\frac{mu}{m+m_o}\right)}{1+\left(\frac{mu}{m+m_o}\right)^2 \frac{1}{c^2}}$$

$$1+\left(\frac{mu}{m+m_o}\right)^2 \frac{1}{c^2} = 2\left(\frac{m}{m+m_o}\right)$$



Simplifying:

$$(m+m_o)^2 + m(\beta)^2 = 2m(m+m_o)$$

$$m_o{}^2 = m^2(1-\beta^2) --->$$

$$m = \frac{m_o}{\sqrt{1-\beta^2}} = m = \gamma m_o$$