Purpose: Practice in simplifying rational expressions and in performing operations with rational expressions and in solving rational equations. To develop understanding of the differences and similarities between these various processes.

Learning Goals: Symbolic and numerical representation; quantitative reasoning; communication.

- Simplify rational expressions; know how to tell a rational expression is in simplest form.
- Multiply or divide rational expressions and write the result in simplest form.
- Add or subtract rational expressions with like or unlike denominators and write the result in simplest form.
- Solve rational equations
- Describe the processes for simplifying, multiplying, dividing, and adding and subtracting rational expressions.
- Describe the process for solving rational equations
- Distinguish between rational expressions and equations.
- Distinguish between the methods used to simplify rational expressions, perform rational operations and solve rational equations

A. SIMPLIFYING RATIONAL EXPRESSIONS. Simplify each rational expression. Show all steps of your work/thinking. NO CREDIT for answers without supporting work.

1. \[
\frac{14x^5}{7x^3 - 28x^3}
\]

2. \[
\frac{x^2 - 7x + 12}{x^2 - 16}
\]

B. MULTIPLYING RATIONAL EXPRESSIONS. Multiply the rational expressions and write the result in simplest form. Show all steps of your work/thinking. NO CREDIT for answers without supporting work.

1. \[
\frac{x^2 - 3x - 10}{5x} \cdot \frac{15x^2}{3x - 15}
\]
C. **DIVIDING RATIONAL EXPRESSIONS.** Divide the rational expressions and write the result in simplest form. Show all steps of your work/thinking. **NO CREDIT** for answers without supporting work.

1. \[
\frac{2m^2 - 5m - 7}{4m^2 - 9} \div \frac{5m^2 + 5m}{4m^2 + 6m}
\]

D. **ADDING AND SUBTRACTING RATIONAL EXPRESSIONS.** Perform the indicated addition or subtraction of rational expressions and write the result in simplest form. Show all steps of your work/thinking. **NO CREDIT** for answers without supporting work.

1. \[
\frac{x^2 + 6x + 7x + 40}{x + 5} + \frac{x + 5}{x + 5}
\]

2. \[
\frac{5a - 12}{a^2 - 8a + 15} - \frac{3a - 2}{a^2 - 8a + 15}
\]
3. \[ \frac{m}{m-2} + \frac{4}{m+2} \]

4. \[ \frac{y}{y+2} - \frac{4}{y^2 + 2y} \]

E. **SOLVING RATIONAL EQUATIONS.** Solve each rational equation and check your solution(s). Indicate any extraneous solutions. Show all steps of your work/thinking. **NO CREDIT** for answers without supporting work.

1. \[ \frac{3x-1}{4x} + \frac{2}{3x} = \frac{7}{6x} \]  

   **Check:**
2. \[2 + \frac{3}{a-3} = \frac{a}{a-3}\]

Check:

F. **EXPRESSIONS vs. EQUATIONS**

1. Describe in your own words the key steps for carrying out each of the following processes:
   a. The process of simplifying a rational expression

   b. The process of multiplying and/or dividing rational expressions

   c. The process of adding and/or subtracting rational expressions

   d. The process of solving a rational equation.
2. Referring to the four processes stated in problem 1,
   a. In which of the four processes is it necessary to find the lowest common denominator (LCD)?

   b. How is the LCD used when adding or subtracting rational expressions?

   c. How is the LCD used when solving rational equations?

3. In each problem below, indicate whether the mathematical statement in the problem is an expression or an equation. If it is an expression, carry out any indicated operations and simplify, if possible. If it is an equation, find all solutions, if any, and indicate any extraneous solutions.

   (a) expression or equation
   \[ \frac{y^2 + 3y}{9} \cdot \frac{3y}{y+3} \]

   (b) expression or equation
   \[ \frac{2}{x-1} + \frac{x-2}{3} = \frac{4}{x-1} \]
(c) \[ \frac{r^2 + 2r}{r - 5} = \frac{5r + 10}{r - 5} \]

(d) \[ \frac{x^2 + 7x + 10}{3x + 6}, \frac{6x - 6}{x^2 + 2x - 15} \]

(e) \[ \frac{x^2}{x + 1} + 2 = \frac{1}{x + 1} \]

(f) \[ \frac{x^2 - 4}{3x + 15} + \frac{x^2 - x - 2}{x^2 + 5x} \]
(g) expression or equation
\[
\frac{1}{c+6} + \frac{4}{c^2+8c+12}
\]

(h) expression or equation
\[
\frac{3}{m-1} - \frac{1}{m+9} = \frac{18}{m^2+8m-9}
\]