Properties of Matrix Addition and Scalar Multiplication

If ***A***, ***B***, and ***C*** are ***m* x *n*** matrices and ***c*** and ***d*** are scalars, then the following properties are true

$A+B=B+A$ Commutative Property of Addition

$$A+(B+C)=(A+B)+C$$
$$=A+B+C$$
Associative Property of Addition

$(cd)A=c(dA)$ Scalar multiplication is both associative and …

$=(dc)A=d(cA)$ commutative

$1A=A$ 1 is the scalar multiplicative identity

Properties of Matrix Addition and Scalar Multiplication

Scalar multiplication distributes over matrix addition

$$c(A+B)=cA+cB$$

Matrix multiplication distributes over scalar addition

$$(c+d)A=cA+dA$$

Properties of Zero Matrices

If ***A*** is an ***m* x *n*** matrix and ***c*** is a scalar, then the following properties are true

$A+O_{mn}=A$ The zero matrix acts as the *additive identity*

$A+(-A)=O_{mn}$ The matrix –*A* is the *additive inverse* of *A*

The third property is similar to the algebra property which states that if the product of two real numbers is zero, then at least one of the numbers must be zero.

If $cA=0_{mn}$, then either $c = 0$, or $A=O_{mn}$.

Properties of Matrix Multiplication

If ***A***, ***B***, and ***C*** are matrices (with sizes such that the given matrix products are defined) and ***c*** is a scalar, then the following properties are true:

$A(BC)=(AB)C$ Associative property of multiplication

$A(B+C)=AB+AC$ Left distributive property

$(B+C)A=BA+CA$ Right distributive property

$c(AB)=(cA)B=A(cB)$ Multiplication by a scalar is both associative and commutative

Properties of the Identity Matrix

If ***A*** is a matrix of size ***m*** x ***n***, then the following properties are true:

$$AI_n = A \qquad I_m A = A$$

Multiplying a matrix by the appropriately sized identity matrix is like multiplying a real number by 1. The identity matrix is the *multiplicative identity* for matrix multiplication.

Properties of Transposes

If ***A*** and ***B*** are matrices (with sizes such that the given matrix operations are defined) and ***c*** is a scalar, then the following properties are true:

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$ The transpose of a sum is the sum of the transposes

$$(cA)^T = c(A^T)$$ You can factor a scalar out of a transpose

$$(AB)^T = B^T A^T$$ The transpose of a product is the product of the transposes in reversed order

Properties of Inverse Matrices

If A is an invertible matrix, k is a positive integer, and c is a scalar, then A^{-1}, A^k, cA, and A^T are invertible and the following are true:

$$(A^{-1})^{-1} = A$$

$$(A^k)^{-1} = \underbrace{A^{-1}A^{-1}A^{-1}\cdots A^{-1}}_{\text{k factors}} = (A^{-1})^k$$

$$(cA)^{-1} = \frac{1}{c}A^{-1},\ c \neq 0$$

$$(A^T)^{-1} = (A^{-1})^T$$