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Math 7 Elementary Linear Algebra

Lab Exercise No. 1

AN ILL-CONDITIONED SYSTEM

In this lab assignment you will examine the effect of round-off error on the solution of a system of linear equations. You will use MatLab to solve and graph the systems. You can use your calculator to assist with computations of slope and *y*-intercepts. Write your solutions and responses on separate paper and hand in.

1. Suppose an experiment leads to the following system of linear equations:

$$\begin{aligned} 4.5x &+ 3.1y &&= 19.249 \\ 1.6x &+ 1.1y &&= 6.843 \end{aligned}$$

   Solve the system by using MatLab to put the augmented matrix of the system in reduced row echelon form. Write down the reduced matrix and the solution of the system.

2. Now solve this system again, but this time with the constants rounded to the nearest hundredth (you'll have to re-enter the matrix).

   Write down the reduced matrix and the solution of the system.

3. Compare your solutions from problems one and two. What happened when the right side of the system was rounded to the nearest hundredth in problem two? What is the percentage change in the right-hand sides from problem 1 to problem 2? What is the percentage change in the calculated values of *x* and *y*? What effect did rounding the constants have on the solution?

4. Examine the system graphically. Graph the pair of lines from problem 1 using the domain $[-1,5]$. Print your graph. What does the graph appear to show? Use zoom and pan to explore the graph, moving from the left-hand endpoint of the domain to the point of intersection. What do you observe? Calculate the slopes and *y*-intercepts of the two lines and write them down. What is the graphical significance of these values?

The system of linear equations in this lab assignment is an example of what is called an ill-conditioned system. Such a system is very sensitive to any changes in the values of its constants. Round-off in the storage of numerical information or during computation can result in a significant change to the solution of the system.