Math 7 Elementary Linear Algebra

Lab Exercise No. 2

APPLICATIONS OF SYSTEMS OF LINEAR EQUATIONS

In this lab exercise you will use MatLab to help you solve applied problems that can be modeled using systems of linear equations. You will be turning in a mixture of hand- and computer-work. Write up your hand-written responses on your own paper. Use "diary" to record your MatLab work. Save it, print it and turn it in with your hand-written work. Please indicate which portions of the diary relate to which problem.

1. **Polynomial Curve Fitting.** For the set of points

$$(0,0),(-1,4.5),(-2,133),(-3,1225.5),(1,-0.5),(2,3),(3,250.5)$$

   a. Write the system of linear equations you would use to find the coefficients of the sixth-degree polynomial that fits the seven points.
   b. Use polyfit to find the coefficients.
   c. Plot the polynomial together with the seven points. Print the plot and turn it in with your solution.

2. **Network Analysis.** Solve Exercise 24 on page 40. Turn in your hand written work showing how you developed the system of linear equations that model the network and giving your answers to all parts of the exercise. Use MatLab to put the augmented matrix of the system into reduced row-echelon form.

3. **Chemical Equations.** Chemical formulas represent the molecular makeup of a chemical compound. For example, when you describe water as H_2O you saying it is comprised of two hydrogen atoms and an oxygen atom. When compounds are combined, their atoms rearrange to form new compounds. For example, burning methane (CH_4) and oxygen (O_2) react to form carbon dioxide (CO_2) and water. The chemical equation which describes this process is given by:

   $CH_4 + O_2 \rightarrow CO_2 + H_2O$ (in chemistry an arrow is used instead of an equal sign)

   There is a problem with the previous equation. Notice that there are two oxygen atoms on the left hand side and three on the right. Also, there are four hydrogen atoms on the left and two on the right. A chemical equation is said to be balanced if the same number of atoms appear on the left as on the right. So, to balance the previous equation, you would introduce coefficients to yield:

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

   Notice that

$$2CH_4 + 4O_2 \rightarrow 2CO_2 + 4H_2O$$

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Is also balanced, but the convention is to use the smallest integers possible. How does one find them? The problem can be solved using a system of linear equations, as shown in the following example.

Example: Hydrochloric acid (*HCl*) and sodium phosphate (Na_3PO_4) react to form phosphoric acid (H_3PO_4) and sodium chloride (*NaCl*). Write a balanced corresponding chemical equation.

Solution: The unbalanced chemical equation is

$$HCl + Na_3PO_4 \rightarrow H_3PO_4 + NaCl$$

Let $x_1, x_2, x_3,$ and x_4 be positive integers which balance this equation; that is, the balanced equation is

$$x_1\left(HCl\right) + x_2\left(Na_3PO_4\right) \rightarrow x_3\left(H_3PO_4\right) + x_4\left(NaCl\right)$$

Equating the number of atoms on both sides of the equation yields:

Hydrogen (*H*)	$x_1 = 3x_3$
Chlorine (*Cl*)	$x_1 = x_4$
Sodium (*Na*)	$3x_2 = x_4$
Phosphorous (*P*)	$x_2 = x_3$
Oxygen (*O*)	$4x_2 = 34x_3$

This gives rise to a system of linear equations

$$\begin{aligned}
x_1 \quad &- \quad 3x_3 \quad &&= \quad 0 \\
x_1 \quad & \qquad\quad - \quad x_4 \quad &&= \quad 0 \\
3x_2 \quad & \qquad\quad - \quad x_4 \quad &&= \quad 0 \\
x_2 \quad &- \quad x_3 \quad &&= \quad 0 \\
4x_2 \quad &- \quad 4x_3 \quad &&= \quad 0
\end{aligned}$$

Observe that this is a homogeneous system, hence either it has only the trivial solution, in which case it is impossible to balance the equation, or it has an infinite number of solutions.

After writing the augmented matrix of the system and reducing it to reduced row-echelon form and solving the resulting system, the general solution of the system is

$$x_1 = t, \quad x_2 = \frac{1}{3}t, \quad x_3 = \frac{1}{3}t, \text{ and } \quad x_4 = t$$

Since we want to find the smallest positive integer solutions, we choose $t = 3$ (*why?*), which yields $x_1 = 3, \quad x_2 = 1, \quad x_3 = 1,$ and $x_4 = 3$ and the balanced equation is

$$3HCl + Na_3PO_4 \rightarrow H_3PO_4 + 3NaCl$$

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Solve the following problem: The chemical reaction below can be used in some industrial processes, such as the production of arsene (AsH_3). Balance the equation.

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$$

Work turned in should include: All hand-written work leading to the system of linear equations, the system of linear equations, MatLab work showing the augmented matrix of the system and the reduced row-echelon form, the general solution of the system, the exact solution which balances the equation using the smallest possible positive integers and the balanced equation.