

Math 7 Elementary Linear Algebra
Lab Exercise No. 3
PROPERTIES OF DIAGONAL MATRICES

In this lab you will explore some properties of matrix operations applied to diagonal matrices.

POWERS OF DIAGONAL MATRICES

Carry out all calculations by hand. DO NOT use a calculator.

For the diagonal matrix

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Use matrix multiplication by hand to calculate D^2 , D^3 , and D^4 . Based on your results, make a conjecture about D^n .

INVERSES OF DIAGONAL MATRICES

1. Find the inverse of each of the diagonal matrices below by using the method of adjoining the identity matrix to another matrix and performing Gauss-Jordan elimination. Carry out all calculations by hand. DO NOT use a calculator to find the inverse.

(a) $A = \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

2. Matrices A and B are diagonal matrices. What appears to be the relationship between the original matrices and their inverses? Make a conjecture about the inverse of a diagonal matrix.

3. Based on your conjecture, what should the inverse of the matrix

$$C = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -12 \end{bmatrix} \text{ be? Use a calculator to verify your result.}$$

4. Using your calculator, find the inverse, if it exists, of the matrix $E = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}$.

What happened? Does your conjecture apply to this matrix? Why or why not?

5. Under what conditions will a diagonal matrix have an inverse? If the diagonal matrix is invertible, what is its inverse? State a formal conjecture about the inverse of an invertible diagonal matrix and prove your conjecture (HINT: To prove two matrices A and B are inverses, show $AB = I = BA$.)