Math 7 Elementary Linear Algebra
STUDY GUIDE FOR EXAM I

GENERAL INFORMATION

Exam Date: Thursday, February 16
This exam will cover Chapter 1, sections 1 and 2, Chapter 2, sections 1 – 4, and Chapter 3, sections 1 – 3.

Unless noted otherwise, you are responsible for all material covered in the text or in lecture. The exam will be given in two parts: in Part I a calculator may not be used; in Part II a calculator may be used. **At no time during the exam may you use a computer and computers must be shut down during the exam.** You may bring a $3" \times 5"$ card of notes; only one side of the card may be used and it must be hand-written. Write your name on the unused side of the card and turn it in with your exam.

EXAM SYLLABUS

Important Definitions: You should know the following definitions. You should be able to explain what they mean, to recognize examples that illustrate them or to apply them.

Chapter One

Consistent system	Inconsistent system	Gaussian elimination
Gauss-Jordan elimination	Row-echelon form	Reduced row-echelon form
Homogeneous linear system		

Chapter Two

Matrix equality	Matrix addition	Scalar multiplication
Matrix multiplication	Identity matrix of order n	Transpose of a matrix
Diagonal matrix	Symmetric matrix	Invertible (nonsingular)
Inverse of a matrix	Singular matrix	Elementary matrix
Row-equivalent	Upper triangular	Lower triangular
LU-factorization		

Chapter Three

Determinant of a 2×2 matrix	Minor	Cofactor

Important Results: You should be familiar with these important results and theorems. You will not be asked to state them, but you should be able to apply them.

Chapter One

Number of solutions of a system of Linear Equations
Elementary Row Operations Number of Solutions of a Homogeneous system

Chapter Two

Properties of Matrix Addition and Scalar Multiplication

Properties of Zero Matrices
Properties of the Identity Matrix
Uniqueness of the Inverse
Inverse of a Product
Representing Elementary Row Operations
A Property of Invertible Matrices
Properties of Matrix Multiplication
Properties of Transposes
Properties of Inverse Matrices
Cancellation Properties
Elementary Matrices Are Invertible

Chapter Three

Determinant of a Triangular Matrix
Elementary Row Operations and Determinants
Conditions that Yield a Zero Determinant
Determinant of a Scalar Multiple of a Matrix
Determinant of an Inverse Matrix
Equivalent Conditions Theorem (p. 147)
Determinant of a Matrix Product
Determinant of an Invertible Matrix
Determinant of the Transpose

Important Processes: We have studied a number of processes and operations. These are summarized here.

Chapter One

1. **Without using a calculator:**
   - Perform Gaussian or Gauss-Jordan elimination using elementary row operations
   - Use Gaussian elimination to put a matrix into row-echelon form
   - Use Gauss-Jordan elimination to put a matrix into reduced row-echelon form

2. Solve a system of linear equations by writing the augmented matrix of the system and using one of the following methods:
   - Gaussian elimination with back-substitution
   - Gauss-Jordan elimination

   You should know how to express unique or infinite solutions in correct form (use parametric form when appropriate) and how to recognize when no solution exists. Identify a system of linear equations as consistent or inconsistent.

3. Use a calculator to
   - Put a matrix into row-echelon form;
   - Put a matrix into reduced row-echelon form;
   - Perform elementary row-operations.

4. Discuss the possible solutions of a general system of linear equations or of a homogeneous linear system.

Recommended Chapter 1 Review Exercises (DO NOT use a calculator unless specifically directed to do so.): 15, 21, 25 – 28 ALL, 29, 31 – 37 ODD without using a calculator (use Gauss-Jordan elimination on at least two of the systems), 43, 47, 49, 51, 59, 60, 61

Chapter Two

1. **Without using a calculator**, perform the operations of matrix addition and subtraction, scalar multiplication of a matrix and matrix multiplication.

2. Use the Properties of Matrix Addition and Scalar Multiplication and the Properties of Matrix Multiplication to carry out matrix operations.

3. Find the transpose of a matrix. Use the Properties of Transposes to manipulate expressions involving transposes of matrices and to perform matrix operations involving transposes.

4. Determine of a matrix is symmetric and explain how you know it is symmetric (or not).

5. Use the definition of an Inverse of a Matrix to determine if one matrix is the inverse of another.

6. Find the inverse of a matrix A, if it exists, by adjoining A and the appropriately sized identity matrix and performing Gauss-Jordan elimination.

7. Use the formula $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ to find the inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Use the definition of an Inverse of a Matrix to prove that this formula gives the inverse of A.

8. Use Theorem 2.9 to find the inverse of a product.

9. Find the inverse of a matrix by using the inverse key on a calculator.

10. Use the inverse of the matrix A to solve the system of linear equations $\mathbf{Ax} = \mathbf{b}$. You may use your calculator to find the inverse.

11. Find the elementary matrix for a specified elementary row operation and find the inverse of the elementary matrix.

12. Express a matrix as a product of elementary matrices.

13. Find an $\boldsymbol{LU}$-factorization of a matrix. Given an $\boldsymbol{LU}$-factorization of a matrix A, use it to solve $\mathbf{Ax} = \mathbf{b}$.

14. Be able to prove the following Theorems or parts of theorems:
    - Theorem 2.1, all parts
    - Theorem 2.2, parts 1 and 2
    - Theorem 2.8, parts 3 and 4
    - Theorem 2.10, all parts

Recommended Chapter 2 Review Exercises (DO NOT use a calculator unless specifically directed to do so.): 1 – 5 ODD, 9, 13, 15 – 29 ODD, 33, 35, 41(a), 44, 45, 46, 47, 49 – 52 ODD, 53

Chapter Three

1. Find the determinant of a 2×2 matrix by using the definition of a 2×2 determinant.

2. Given a matrix A, find a specified minor or cofactor of A.

3. Use expansion by cofactors to find the determinant of a matrix.

4. Use a calculator to find the determinant of a matrix.

5. Use Theorem 3.2 to find the determinant of a triangular matrix.

6. Evaluate $|A|$ by first using elementary row operations or elementary column operations to rewrite A in triangular form and then apply Theorem 3.2.

7. Use Properties of Determinants to calculate the determinant of
   - a product of matrices
   - a scalar multiple of a matrix
   - the inverse of a matrix
   - the transpose of a matrix.

8. Use the **Equivalent Conditions Theorem** to determine whether a system of linear equations has a unique solution by calculating the determinant of the coefficient matrix of the system.

Recommended Chapter 3 Review Exercises (DO NOT use a calculator unless specifically directed to do so.): 5, 7, 11, 23, 25, 27, 37, 39, 41, 43, 44