Factoring Trinomials by Grouping

The process of factoring by grouping can be used to factor trinomials of the form \( ax^2 + bx + c \). For instance, suppose you had the trinomial \( 6x^2 - 19x + 10 \). This trinomial can be re-written and factored by grouping as follows:

\[
6x^2 - 19x + 10 = 6x^2 - 4x - 15x + 10 \quad \text{re-write } -19x \text{ as } -4x - 15x
\]

\[
= (6x^2 - 4x) - (15x - 10) \quad \text{group the first two terms And the last two terms, factoring out the negative sign in the second grouping}
\]

\[
= 2x(3x - 2) - 5(3x - 2) \quad \text{factor out the GCF from each group}
\]

\[
= (3x - 2)(2x - 5) \quad \text{factor out the common group factor}
\]

The result is the binomial factorization of the original trinomial: \( 6x^2 - 19x + 10 = (3x - 2)(2x - 5) \).

The trick is to split the middle term (\(-19x\) in the example above) into the sum or difference of two terms so that the resulting 4-term polynomial can be factored by grouping. Here’s how you do it.

To factor \( ax^2 + bx + c \), first find two factors of \( a \cdot c \) whose sum is \( b \). Use the factors to split the middle term and then factor by grouping.

Here are two examples.

Example 1. Factor \( 2x^2 + 13x + 15 \).

**Step 1:** Identify \( a, b \) and \( c \): \( a = 2, b = 13, c = 15 \).

**Step 2:** Multiply together \( a \) and \( c \): \( a \cdot c = 2 \cdot 15 = 30 \)

**Step 3:** Since the sign of the middle term is positive, find two positive factors of 30 whose sum is 13 (the middle term coefficient).

<table>
<thead>
<tr>
<th>Positive Factors of 30</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 30</td>
<td>31</td>
</tr>
<tr>
<td>2 and 15</td>
<td>17</td>
</tr>
<tr>
<td>3 and 10</td>
<td>13</td>
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</tbody>
</table>

\[ \Rightarrow \text{This is the required sum. When the required sum has been found, the remaining factors need not be checked.} \]

**Step 4:** Use the factors of 30 whose sum is 13 to write 13x as 3x + 10x and factor by grouping.

\[
2x^2 + 13x + 15 = 2x^2 + 3x + 10x + 15 \quad \text{re-write } 13x \text{ as } 3x + 10x
\]

\[
= (2x^2 + 3x) + (10x + 15) \quad \text{group the first two terms and the last two terms}
\]

\[
= x(2x + 3) + 5(2x + 3) \quad \text{factor out the GCF from each group}
\]

\[
= (2x + 3)(x + 5) \quad \text{factor out the common group factor}
\]
Example 2. Factor $6x^2 - 11x - 10$.

**Step 1:** Identify $a$, $b$ and $c$: $a = 6$, $b = -11$, $c = -10$.

**Step 2:** Multiply together $a$ and $c$: $a \cdot c = 6 \cdot (-10) = -60$

**Step 3:** Find two factors of $-60$ whose sum is $-11$ (the middle term coefficient). Note that this time one of the factors will be positive and the other will be negative because the constant term is negative.

<table>
<thead>
<tr>
<th>Factors of $-60$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and $-60$</td>
<td>$-59$</td>
</tr>
<tr>
<td>$-1$ and $60$</td>
<td>$59$</td>
</tr>
<tr>
<td>2 and $-30$</td>
<td>$-28$</td>
</tr>
<tr>
<td>$-2$ and $30$</td>
<td>$28$</td>
</tr>
<tr>
<td>3 and $-20$</td>
<td>$-17$</td>
</tr>
<tr>
<td>$-3$ and $20$</td>
<td>$17$</td>
</tr>
<tr>
<td>4 and $-15$</td>
<td>$-11$</td>
</tr>
</tbody>
</table>

⇐ This is the required sum. When the required sum has been found, the remaining factors need not be checked.

**Step 4:** Use the factors of $-60$ whose sum is $-11$ to write $-11x$ as $-15x + 4x$ and factor by grouping.

\[
6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10 \quad \text{re-write } -11x \text{ as } -15x + 4x
\]

\[
= (6x^2 - 15x) + (4x - 10) \quad \text{group the first two terms and the last two terms}
\]

\[
= 3x(2x - 5) + 2(2x - 5) \quad \text{factor out the GCF from each group}
\]

\[
= (2x - 5)(3x + 2) \quad \text{factor out the common group factor}
\]

Now you try one.

**Factor** $12y^2 - 7y + 1$. Note that the constant term is positive and the middle term is negative, so both factors of $a \cdot c$ will be negative.