Factoring Trinomials by Grouping

The process of factoring by grouping can be used to factor trinomials of the form $ax^2 + bx + c$.

For instance, suppose you had the trinomial $6x^2 - 19x + 10$. This trinomial can be re-written and factored by grouping as follows:

$6x^2 - 19x + 10$	$=$	$6x^2 - 4x - 15x + 10$	re-write $-19x$ as $-4x - 15x$
	$=$	$(6x^2 - 4x) - (15x - 10)$	group the first two terms And the last two terms, factoring out the negative sign in the second grouping
	$=$	$2x(3x - 2) - 5(3x - 2)$	factor out the GCF from each group
	$=$	$(3x - 2)(2x - 5)$	factor out the common group factor

The result is the binomial factorization of the original trinomial: $6x^2-19x+10 = (3x-2)(2x-5)$.

The trick is to split the middle term ($-19x$ in the example above) into the sum or difference of two terms so that the resulting 4-term polynomial can be factored by grouping. Here's how you do it.

To factor $ax^2 + bx + c$, first find two factors of $a \cdot c$ whose sum is b. Use the factors to split the middle term and then factor by grouping.

Here are two examples.

Example 1. Factor $2x^2 + 13x + 15$.

Step 1: Identify a, b and c: $a = 2$, $b = 13$, $c = 15$.

Step 2: Multiply together a and c: $a \cdot c = 2 \cdot 15 = 30$

Step 3: Since the sign of the middle term is positive, find two positive factors of 30 whose sum is 13 (the middle term coefficient).

Positive Factors of 30	Sum	
1 and 30	31	
2 and 15	17	
3 and 10	13	$\Leftarrow$ This is the required sum. When the required sum has been found, the remaining factors need not be checked.

Step 4: Use the factors of 30 whose sum is 13 to write $13x$ as $3x+10x$ and factor by grouping.

$2x^2 + 13x + 15$	$=$	$2x^2 + 3x + 10x + 15$	re-write $13x$ as $3x + 10x$
	$=$	$(2x^2 + 3x) + (10x + 15)$	group the first two terms and the last two terms
	$=$	$x(2x + 3) + 5(2x + 3)$	factor out the GCF from each group
	$=$	$(2x + 3)(x + 5)$	factor out the common group factor

Here is an example with negative terms.

Example 2. Factor $6x^2 - 11x - 10$.

Step 1: Identify a, b and c: $a = 6$, $b = -11$, $c = -10$.

Step 2: Multiply together a and c: $a \cdot c = 6 \cdot (-10) = -60$

Step 3: Find two factors of -60 whose sum is -11 (the middle term coefficient). Note that this time one of the factors will be positive and the other will be negative because the constant term is negative.

Factors of -60	**Sum**	
1 and -60	-59	
-1 and 60	59	
2 and -30	-28	
-2 and 30	28	
3 and -20	-17	
-3 and 20	17	
4 and -15	-11	$\Leftarrow$ This is the required sum. When the required sum has been found, the remaining factors need not be checked.

Step 4: Use the factors of -60 whose sum is -11 to write $-11x$ as $-15x + 4x$ and factor by grouping.

$6x^2 - 11x - 10$	$=$	$6x^2 - 15x + 4x - 10$	re-write $-11x$ as $-15x + 4x$
	$=$	$(6x^2 - 15x) + (4x - 10)$	group the first two terms and the last two terms
	$=$	$3x(2x - 5) + 2(2x - 5)$	factor out the GCF from each group
	$=$	$(2x - 5)(3x + 2)$	factor out the common group factor

Now you try one.

Factor $12y^2 - 7y + 1$. Note that the constant term is positive and the middle term is negative, so both factors of $a \cdot c$ will be negative.