Refraction at plane surfaces

- Speed of light in air
  - Slight digression on what is light
  - Fizeau’s measurement
- Index of refraction
  - speed of light in transparent materials
  - dispersion
  - wave picture of refraction
  - ray picture, Snell’s law
- Plane refractive surfaces
  - single surfaces, images
  - plane-parallel plate
- Critical angle
- Prisms
What is light?

- Carries energy
- Exerts force on electrical charges
- Wave similar to sound or water wave
  - Frequency related to color (high frequency=blue, low frequency = red)
  - Wavelength also related to color (long wavelength=red, short wavelength=blue)
  - Other electromagnetic radiation behaves similarly (radio, microwave, x-rays, gamma rays)
- Wave fronts perpendicular to rays

Wavefront – surface of peaks of waves

Wave speed, V

Wave length, $\lambda$

amplitude
How fast does light travel

• Light travels in a straight line
  – How fast does it travel?

• Galileo’s method (1600)
  – Lantern’s on neighboring hills
  – One person unblocks his lantern
  – When the other sees the lantern, he unblocks his
  – First person sees light from the second lantern, giving round trip time
  – Doesn’t work, light too fast
Measurement of speed of light in air

- Olaf Romer (1675) used eclipses of Jupiter’s moons estimated
  \( c = 2.967 \times 10^{10} \text{cm/sec} \)
- Fizeau (1849) first earthbound measurement

Speed of light is constant
- Independent of color (wavelength)
- Independent of motion of source

Symbol, \( c \) (Latin, \textit{celere}=speedy)
Speed of light in transparent media

• First measured by Foucault in 1850
  – helped to settle controversy about wave/particle nature of light
  – light travels slower in denser media

• Be careful about the meaning of the word “density”

\[ \text{index of refraction} = \frac{\text{speed in vacuum}}{\text{speed in medium}} \]
Dispersion

Index of refraction of Quartz

- Index of refraction (speed) changes with wavelength
- Decreases for longer wavelengths
- Small, but important effect, note y-axis doesn’t start at zero
Refractive index also varies widely between materials

Index of refraction of various materials at 589 nm (d light)

<table>
<thead>
<tr>
<th>Material</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.333</td>
</tr>
<tr>
<td>Air</td>
<td>1.000293</td>
</tr>
<tr>
<td>BK7 glass (crown)</td>
<td>1.517</td>
</tr>
<tr>
<td>BaSF6 glass (flint)</td>
<td>1.668</td>
</tr>
<tr>
<td>FDS9 glass</td>
<td>1.847</td>
</tr>
<tr>
<td>Quartz</td>
<td>1.458</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.419</td>
</tr>
</tbody>
</table>
Refraction of a wavefront at plane surface

- Wavelength in higher index material is shorter
  - velocity slower but frequency unchanged
- Wavefronts are continuous across surface
- Wave bends towards the normal when going to a higher index material
Refraction at a plane surface-ray picture

Snell’s law

**part 1, ray angle**

\[ n \sin(\Theta) = n' \sin(\Theta') \]

or

\[ \sin(\Theta') = \frac{n}{n'} \sin(\Theta) \]

**part 2, ray orientation**—incident ray, reflected ray, and surface normal all lie in the same plane

\( n'/n \) is called the relative refractive index

Not all the energy is refracted, some is reflected. Snell’s law doesn’t say how much is reflected.
Graphical construction for refraction

- Set a compass to radius $n_1$ (units don’t matter!!!)
  - Use this to mark off a distance $n_1$ along input ray from point of intersection with surface
  - This is point A on the diagram
- Set compass to $n_2$
  - With point at A draw arc that intersects surface normal
  - Intersection point is B in diagram
- Refracted ray is parallel to line AB
  - Transfer a parallel line to the intersection point
Principle of reversibility

- If a ray is reversed in direction:
  - output angle $\Theta_2$ becomes input angle
  - final index, $n_2$ becomes initial index
  - angle which satisfies Snell’s law for the final angle is then $\Theta_1$, the original input angle
  - therefore, the ray retraces its path

- The law of reflection is symmetrical with respect to its initial and final directions also

In geometrical optics, if any ray is reversed it will retrace its original path. This holds for all rays.
Refraction of rays from a point source

- Rays at a higher angle bend more
- From in the high index material, light appears to come from an image point farther from the surface than the object
  - if \( n_2 < n_1 \) then the image is closer to the surface
- The image is virtual
Refraction of rays from a point source

Snell’s law: \( \sin(\Theta) = \frac{n'}{n} \sin(\Theta') \)

For small angles this is approximately \( \Theta = \frac{n'}{n} \Theta' \)

(paraxial approximation angle in radians)

In this approximation \( h' = \frac{n'}{n} h \)

Exact result
\[
\frac{h'}{\cos(\Theta')} = \frac{n'}{n} \frac{h}{\cos(\Theta)}
\]

Not a perfect image, location depends on ray angle!
Pencil appears to bend at water surface

- Light rays from pencil tip refract at water surface, before entering camera lens; appear to come from “bent” pencil

Questions: If you want to touch the pencil tip with another pencil tip, should you aim for the actual tip location or the apparent tip location?

If you want to shine a laser beam at the tip, do you aim it at the apparent or the actual location?
Rays out of plane give different image from in plane rays – astigmatism

- Observer will see both sets of rays, therefore blurry image
- Only meridional rays appeared on previous diagram
Refraction through a plane-parallel slab

- final ray parallel to the initial ray
  - not true if faces are not parallel
  - displacement, \( d \), proportional to thickness, \( t \), increases with larger index

Geometry gives:

\[
d = h \sin(\phi_1 - \phi_2) = t \frac{\sin(\phi_1 - \phi_2)}{\cos(\phi_2)}
\]

This can be expressed in different ways using trigonometry

\[
d = t \left[ \sin(\phi_1) - \frac{\sin(\phi_2) \cos(\phi_1)}{\cos(\phi_2)} \right] = t \sin(\phi_1) \left[ 1 - \frac{n_1 \cos(\phi_1)}{n_2 \cos(\phi_2)} \right] = t \sin(\phi_1) \left[ 1 - \frac{\cos(\phi_1)}{\sqrt{n_2^2 - \sin^2(\phi_1)}} \right]
\]

Last form doesn’t require calculation of \( \phi_2 \)
Imaging or focusing through a parallel plate

- Imaging – object located at O, appears to be at O’
  - Application, microscope imaging through cover glass
- Focusing – light to left of plate is focusing towards O’
  - Actually focuses at O
  - Application, machining laser focused through a window, for example to prevent debris from getting on lens
- Imaging or focusing depending on direction of travel
  - reversibility

$$E = t \frac{n - 1}{n}$$
Prisms

- Deflections at the two surfaces don’t cancel
- Deflection angle depends on apex angle of prism, $\alpha$, not angles of base
- From geometry $\delta = \phi_1 + \phi_2' - \alpha$
  - can be considered a function of $\phi_1$ since $\phi_2$ depends on $\phi_1$ and $\alpha$ is fixed for a given prism
  - depends on index of refraction and therefore also on wavelength
Graphical construction for deviation by a prism

• Begin by finding the refraction at the first surface as before
  – Before setting compass for n₂, draw an arc of radius n₁ centered on A

• Through the point B draw a line perpendicular to the second surface
  – the point where this line intersects the n₁ circle drawn previously is C

• The final ray exiting the prism is parallel to AC
Deflection of a prism as a function of incidence angle

- Minimum deviation depends only on index and prism apex angle

Deflection angle of an equilateral prism with an index of 1.517
Minimum deviation formulas

- Formulae derived simply by setting $\phi_1 = \phi_2'$ and $\phi_1' = \phi_2$
- Obviously, $\phi_1'$ can be found from $\phi_1$ from Snell’s law
- The details of the derivation are not too important, but not too difficult either

**Final results VALID ONLY FOR MINIMUM DEVIATION**

To find input angle at minimum deviation angle given index and apex angle

$$\phi_1 = \sin^{-1}\left(n \sin\left(\frac{A}{2}\right)\right)$$

To find minimum deviation angle, given index and apex angle

$$\delta_{\text{min}} = 2\phi_1 - A$$

To find index, given minimum deviation angle and apex angle

$$n = \frac{\sin\left(\frac{\delta_{\text{min}} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$
Critical angle, total internal reflection

- A ray traveling from a higher index to a lower index bends away from the normal
  - at an angle called the critical angle, the refracted angle is 90°, the ray travels along the surface
  - for larger incidence angles, Snell’s law says the sine of the refracted angle is greater than one, this is impossible

\[ \sin(\Theta_{\text{critical}}) = \frac{n'}{n} \]

- For incident angles larger than the critical angle, the light is completely reflected, there is no refracted ray
Common prisms using total internal reflection

- Total reflection
- Porro
- Dove or inverting
- Amici or roof
- Triple mirror
- Lummer-Brodhun