Imaging with two lenses

• Graphical methods
  – Parallel-ray method, find intermediate image, use as object for next lens
  – Virtual objects
  – Oblique-ray method, lens-to-lens, no need to find intermediate image

• Mathematical methods
  – Find intermediate image, use as object for next lens
  – lens-to-lens (sequential raytracing)-later in semester

• Combinations of thin lenses
  – In contact
  – Separated
Example, two separated positive lenses

- Needed information
  - focal lengths of lenses
  - location of lenses
  - location of object
Parallel-ray method - step 1

- Ignore second lens
- Trace at least two of the rays shown from tip of object
  - tip of image found from intersection of rays
  - exactly as we did in previous module
  - NOTE: all rays from tip of object intersect at tip of image we have chosen three only because they are easy to trace!
  - Image is real in this case, but method is exactly the same if it is virtual WHENEVER NEEDED YOU MAY EXTEND OBJECT-SPACE OR IMAGE-SPACE RAYS TO INFINITY IN EITHER DIRECTION
Parallel-ray method - step 2

- Image from step 1 becomes Object for step 2
  - After producing this object, lens 1 one can be ignored
  - Object can be real or virtual (virtual in this case)
  - Object can be real even if image from lens 1 is virtual

- Trace any two of the three rays shown through tip of object to find tip of final image
  - Final image may be real or virtual (real in this case)
Reminder about real and virtual objects and images

Remember: For purposes of calculations, light travels from left to right (not necessarily in real life, of course)

• Positive distances correspond to real objects and real images
  – Object distance positive when it is to the left of the lens
  – Image distance positive when it is to the right of the lens
  – This is the common situation for a single positive image forming an image on a screen, Examples- viewgraph machine, camera, eye, etc.

• Negative distances correspond to virtual objects and virtual images
  – Object to the right or image to the left of the lens
Oblique-ray method

- Is it really necessary to find the image due to the first lens?
- Any ray can be traced through the lens system using the oblique-ray method
- For example, trace the axial ray
  - point of intersection with axis in image space gives image location
Imaging through multiple lenses - mathematical

Given, $f_1, f_2, d,$ and $s_1$ find $s_2'$. 

- Apply imaging formula to first lens to find image in that lens
  $$s_1' = \frac{s_1 f_1}{s_1 - f_1}$$
- Find object distance for second lens (negative means virtual object)
  $$s_2 = d - s_1'$$
- Use imaging formula again to find final image
  $$s_2' = \frac{s_2 f_2}{s_2 - f_2}$$
Magnification with multiple lenses

- The magnification is by definition the image size divided by the object size.

- For the second lens in a system the object size is the image size for the first lens.

\[ y_2 = y_1' = M_1 y_1 \]

- The image size after the second lens is found by multiplying the second lens magnification by the size of the object for the second lens.

\[ y_2' = M_2 y_2 = M_1 M_2 y_1 \]

- The system magnification is the final image size divided by the original object size.

\[ M_{\text{system}} = M_1 M_2 \]
Symbols for thin lenses

• Arrows symbolize the outline of the glass at the edge
Raytracing through several lenses

• Go through lenses in the order the light strikes them
  – This is true even if there are mirrors in the system!
• For mathematical raytracing be careful of sign convention
  – many possibilities of focal lengths and spacings but sign convention covers them all
Special considerations for negative lenses – there aren’t any

- Note that the prime on focal points for negative lens have reversed
- Primary and secondary focal points on opposite sides compared to positive lens
Principal planes and focal lengths of multi-lens systems

- Defined exactly the same as for thick lens
Thin lenses in contact

\[
\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad P = P_1 + P_2
\]

- Can be applied to multiple thin lenses in contact as well
- As always, be careful with sign convention
Dispersion-Abbe V number

Index of refraction changes with wavelength
- Usually decreases for longer wavelengths
- Small, but important effect

Abbe V number-Definition

\[ V_d = \frac{n_d - 1}{n_F - n_C} \]

For Quartz, \( V_d = 67.6 \)

Index of refraction of Quartz

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<tr>
<th>Wavelength (nm)</th>
<th>Refractive Index</th>
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<tr>
<td>350</td>
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<tr>
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<tr>
<td>650</td>
<td>1.466</td>
</tr>
<tr>
<td>700</td>
<td>1.468</td>
</tr>
</tbody>
</table>

Glass designation
Quartz, 459676
BK7, 517642
Glass map - index/dispersion of glasses
Chromatic aberration of thin lenses

\[ P_F = (n_F - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ P_C = (n_C - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ P_F - P_C = (n_F - n_c) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{(n_F - n_C)P_d}{n_d - 1} = \frac{P_d}{V_d} \]

- Called longitudinal chromatic aberration
  - This is the only chromatic aberration possible in a single thin lens with the stop at the lens
Achromatic doublets

To achieve the desired focal length requires

$$P_d = P_{+d} + P_{-d}$$

Many choices of powers satisfy this equation.

We can also choose the powers to make chromatic zero

$$P_F - P_C = 0 = (P_{+F} - P_{+C}) + (P_{-F} - P_{-C}) = \frac{P_{+d}}{V_{+d}} + \frac{P_{-d}}{V_{-d}}$$

- Two radii used to get each power, one can be freely chosen.
- One is usually chosen to make inner surfaces have the same curvature (cemented doublet).
- Remaining radius chosen to minimize spherical aberration.
Thin lenses separated

- Two positive thin lenses are weaker in combination when separated than when in contact.
- Use of these equations in solving homework problems is discouraged.

\[
\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}
\]

\[
P = P_1 + P_2 - dP_1 P_2
\]

Back focal length

\[
F_B = \left(1 - \frac{d}{f_1}\right) \frac{1}{P} = \left(1 - \frac{d}{f_1}\right) F
\]

Second lens to principal plane: 

\[
F_B - F = -\frac{P_1}{P} d
\]
Principal plane locations for thin lens combinations

- $c_1$ is power of first lens
- $f_2$ is focal length of second lens
- Focal length of combination is kept constant at 100 mm

The position of the principal planes for several lens combinations is illustrated in Fig. 4.10. Many interesting properties may be noticed by a close examination of this figure.