

Newton's Laws and Archimedes's Principle



Purpose:

To determine the densities of four metal cubes through an understanding of Archimedes's Principle.

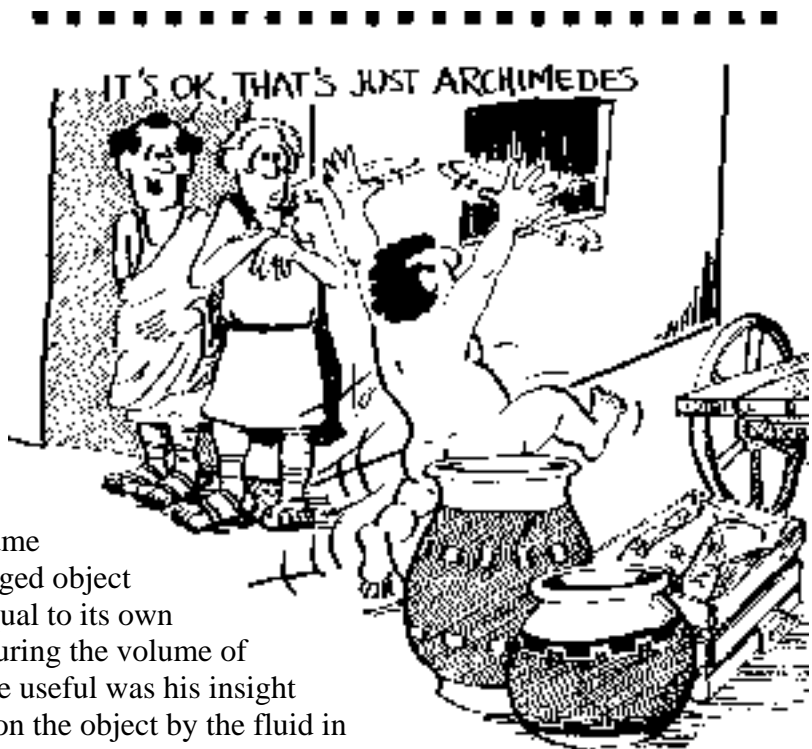
Equipment:

- Hooked Metal Cubes (Aluminum, Steel, Lead and Brass)
- Hooked Mass Set
- Beam Balance
- Beakers
- Rods and Clamps
- String

Theory:

The apocryphal story of Archimedes tells that as he was getting into the bath one night, he noticed that the water level in the tub rose as he got in, and fell when he got out, in an amount proportional to how much of himself was immersed in the water. His *Eureka!* moment came when he realized that a submerged object displaced a volume of water equal to its own volume...an easy way of measuring the volume of irregular objects! Perhaps more useful was his insight into the buoyant force exerted on the object by the fluid in

Cartoon by Ken Otter, UNBC Canada

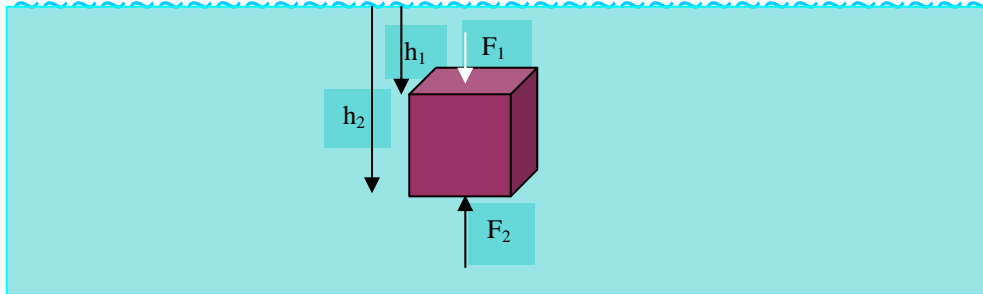


which it is immersed, called *Archimedes' Principle*:

An immersed body is buoyed up by a force equal to the weight of the fluid it displaces.

Let's see how this works:

Imagine there is a cube immersed in water. The length of each side of the cube is l . The density of the water is ρ .



The water exerts pressure on the cube from all sides. Since the vertical sides of the cube are of the same size and at the same depth, it is easy to see that the net force exerted in the horizontal direction is equal to zero. Thus, the total buoyant force on the cube will come from the difference in pressures on the horizontal faces of the cube: the top and the bottom, and will act in the vertical direction.

The pressure exerted by a fluid is dependent upon the density of the liquid and the depth of the object below the surface:

$$\text{Pressure} = \rho gh \quad \text{Eq. 1}$$

The force acting on an object depends on the area over which the pressure is exerted:

$$\text{Force} = \text{pressure} \times \text{area}$$

$$F = \rho ghl^2 \quad \text{Eq. 2}$$

Thus, we have the forces acting on the cube shown above:

$$F_1 = \rho gh_1l^2$$

$$F_2 = \rho gh_2l^2$$

$$\Delta F = F_2 - F_1 = \rho gl^2(h_2 - h_1)$$

$$= \rho gl^3$$

since the difference in depth of the top and bottom of the cube is simply the height of the cube, l .

Going back to Archimedes' *Eureka!* moment, we know that the cube of volume l^3 displaces a volume of water equal to l^3 . Thus, the expression for the buoyant force on the cube is simply equal to the weight of the water displaced:

ρ = the density of water

l^3 = the volume of water displaced

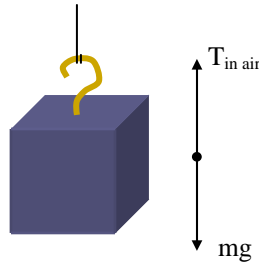
ρl^3 = the mass of water displaced

$\rho l^3 g$ = the weight of water displaced

QED

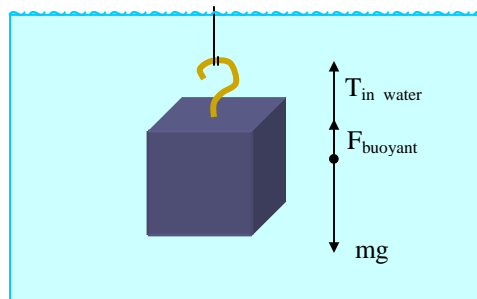
Experiment:

1. Check the zero mark of your balance.
2. Suspend the object whose specific gravity is to be determined from the hook under the base of the left-hand pan. You will need to tie the objects with string. Use as little string as possible.
3. Weigh the object in air by adding weights to the right-hand pan and moving the sliding scale to the right until the scale pans are balanced. When suspended in air, the forces are as follows:



where $T_{in\ air}$ is the tension in the string when the sample is suspended in air, or "weight in air". From the diagram, you can see that $T_{in\ air} = mg = \rho_s Vg$ where V is the volume of sample, and ρ is the density of the sample

4. Weigh each object again, this time while submerged in a beaker of water. When suspended in water, the forces are as follows:



where $T_{in\ water}$ is the "weight-in-water." Thus $T_{in\ water} = mg - F_{buoyant}$. And of course, Archimedes' principle is that the buoyant force equals the weight of the water displaced:

$$F_{buoyant} = \rho_{water} Vg \quad \text{Eq. 3}$$

- Repeat steps 1-4 above for the three other metal cubes.

Analysis:

- Now its up to you to prove that:

$$\frac{\rho_{\text{sample}}}{\rho_{\text{water}}} = \frac{\text{weight} - \text{in} - \text{air}}{(\text{weight} - \text{in} - \text{air}) - (\text{weight} - \text{in} - \text{water})}$$

The ratio $\frac{\rho_{\text{sample}}}{\rho_{\text{water}}}$ is called the *specific gravity* of the sample. The density of the sample $\rho_{\text{sample}} = \text{specific gravity} \times \rho_{\text{water}}$, where $\rho_{\text{water}} = 1.0 \text{ g/cm}^3$.

- Calculate the specific gravity of each sample studied.
- Determine whether the measured densities are equivalent to the known values within the limits of your uncertainties. If not, explain.

Results:

Write at least one paragraph describing the following:

- what you expected to learn about the lab (i.e. what was the reason for conducting the experiment?)
- your results, and what you learned from them
- Think of at least one other experiment might you perform to verify these results
- Think of at least one new question or problem that could be answered with the physics you have learned in this laboratory, or be extrapolated from the ideas in this laboratory.

Clean-Up:

Before you can leave the classroom, you must clean up your equipment, and have your instructor sign below. How you divide clean-up duties between lab members is up to you.

Clean-up involves:

- Completely dismantling the experimental setup
- Removing tape from anything you put tape on
- Drying-off any wet equipment
- Putting away equipment in proper boxes (if applicable)
- Returning equipment to proper cabinets, or to the cart at the front of the room
- Throwing away pieces of string, paper, and other detritus (i.e. your water bottles)
- Shutting down the computer
- Anything else that needs to be done to return the room to its pristine, pre lab form.

I certify that the equipment used by _____ has been cleaned up.

(student's name)

_____, _____.

(instructor's name)

(date)