Double Slit Interference and Single Slit Diffraction

Purpose:
- To determine the wavelength of a laser using double slit interference
- To determine the wavelength of a laser using single slit diffraction
- To explore the effects of interference and diffraction together.

Equipment:
- Optics kit and Optical Bench
- He-Ne Laser
- LabPro Kit
- Light Sensor
- Vernier Calipers
- Red Reading Lights

Theory:
Back in the day, everyone assumed that light was composed of little particles...Newton called them “corpuscles”. This sort of thinking goes all the way back to the Greeks... the Greek philosophers were the first to propose the atomic theory of nature; that everything in and around us is made up of little indivisible pieces of stuff; so it seems natural that they would want to apply this same theory to light as well. Newton was the king of particle mechanics, so it is likewise natural that he would assume particles were the basis of light.

Newton’s justification for a particle theory of light came from observations of shadows around barriers. Imagine tossing baseballs at the edge of a brick wall (no curve balls here!). Some of the balls will hit the wall directly and bounce back in your direction. Some will miss the wall completely and continue traveling in a straight line until they fall to the ground. Some will glance off of the edge of the wall and be deflected slightly away from the wall while keeping most of its initial velocity. After throwing a large number of baseballs, you might see a pattern something like this:

![Figure 1 Baseballs and Shadows](image-url)
Notice how there are no balls behind the wall. This is consistent with the behavior of particles – they don’t “bend” around a barrier. Because Newton and others who came before him observed light making sharp shadows, they assumed that it was because light was of a particle nature, as it was not behaving in a wave-like manner.

Water can form waves, and when those waves meet with an obstruction, they sort of “fan out” around the side:

![Figure 2 Bending Waves](image)

If the obstruction is approximately the same size as the distance between the waves (the wavelength), the “shadow” space is quickly filled in with waves:

![Figure 3 Shadow Filling In](image)

If the obstruction is much larger than the length of the waves, a sharp shadow is cast:

![Figure 4 Sharp Shadows](image)

This bending of waves around the corners of obstructions is called *diffraction*. You can see this effect around breakwaters or in the narrow mouth of a bay. This is also the reason you can hear sounds around the corners of buildings, or when standing behind a tree.
Waves can also interfere, as we already know from the study of standing waves on string and in air. Both of these examples occurred in one dimension, but two- (or three-) dimensional waves will form two- (or three-) dimensional interference patterns.

A series of pebbles dropped into still water at regular intervals will produce a circular pattern of waves:

![Figure 6 Pebble Ripples](image)

where the solid lines (red) represent wave crests and the broken lines (blue) represent wave troughs. If two people next to each other drop pebbles in sync, the patterns of waves will overlap, like this:

![Figure 7 Interfering Ripples](image)

Notice that there are lines where wave crests meet other crests, and troughs meet other troughs:

![Figure 8 Constructive Interference](image)
Notice also that there are lines where crests meet up with troughs:

Figure 9  Destructive Interference

As you already know, in the areas where there crests meet with other crests, there is *constructive* interference, which doubles the amplitude of the wave, and in areas where crests meet with troughs there is *destructive* interference, which cancels out the amplitude of the wave. If we took a snapshot of the interfering ripples, this is what they would look like:

Figure 10

Interfering waves: computer simulation created by Johnny Erickson, Salt Lake Community College. Check out the animation at [http://www.slcc.edu/schools/hum_sci/physics/designers/erickson/](http://www.slcc.edu/schools/hum_sci/physics/designers/erickson/)

The gray fuzzy stripes are areas of total destructive interference. The surface of the water is the same as the undisturbed (equilibrium) water level. The black and white stripes are areas of constructive interference, with total constructive interference (the highest amplitudes) occurring in the centers of the bands.

Looking at the side of the image, we could mark out the points of total constructive and destructive interference:

Figure 11
If we were able to look at a point that was much farther away from the sources, or if the sources were much closer together, the spacing between the points of total constructive and destructive interference would be more equal.

It was this type of interference effect that Thomas Young first noticed in 1800, in an experiment which has been mutated into the classic “Young’s Double Slit” experiment. According to Walter Scheider, the actual experiment that Young performed and then demonstrated for the Royal Society of London in 1803 consisted of sunlight directed into the lecture room by means of a mirror, through a small hole in the window shutter to create a small beam of light which is then incident on a “slip of card,” one thirtieth of an inch in width.

![Diagram of Young's Experiment](image-url)

**Figure 12 Thomas Young’s Experiment**

The slip of card splits the ray of sunlight into two beams, which then interfere. The interference pattern can be projected on a screen. Nowadays, this experiment is performed using a laser or LED as the light source, and a blackened slide on which two parallel slits have been etched (hence, the “double slit experiment”). These modifications both simplify and complicate the original experiment...we’ll get to those in a bit.

First, let’s study why we get an interference pattern at all. We will use sound waves in our example, but the same derivation applies to all waves.

Imagine you have two speakers hooked up to a function generator so that the sound waves coming out have the same wavelength, frequency and phase. If the speakers are side-by-side, and you, the listener, are standing along the line that runs between them, you will hear constructive interference of the waves; that is, a louder volume than with just one speaker.

![Diagram of Sound Waves](image-url)

**Figure 13**

If the speakers are not sitting next to each other, but there is some distance between them, what the listener will hear depends on the path length difference between him and the speakers.

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If the path length from one source to the listener differs by a whole number of wavelengths from the path length of the other source to the listener, the listener will experience total constructive interference. If the path length difference is a whole number plus one-half wavelengths, the listener will experience total destructive interference. The same will be true in two or more dimensions as well.
If we looked at other points of constructive interference, we would find that \( \Delta L = m\lambda \), where \( m \) is an integer, \( \lambda \) is the wavelength, and \( \Delta L \) is the path length difference from the sources. (For points of destructive interference we would find \( \Delta L = (m + \frac{1}{2})\lambda \).)

If we imagine a line that perpendicularly bisects the line connecting the sources (a “normal” of sorts...), we can define the points of constructive interference by their angle away from this line.
If we assume that the distance, \( d \), between the sources is much smaller than the path lengths \( L \), we have the approximation that \( \Delta L = dsin\theta \).

\[
\Delta L = L_B - L_A
\]

**Figure 19**

Remembering that \( \Delta L = m\lambda \), we can write the expression

\[
m\lambda = d\sin\theta.
\]  

**Eq. 1**

This is the equation we use to describe two-dimensional wave interference, whether water, sound, radio, micro-, or visible light waves.

Now that we see that the angle at which constructive interference will occur depends on the wavelength of the wave, we can understand why modern-day “Young” experiments use lasers instead of sunlight, or another white-light source. Sunlight, as well as all other “white” light, is composed of all the different colors of the rainbow (though some colors may be more dominant than others). What distinguishes one color from the next is a different frequency, and therefore a different wavelength. As the sunlight passes through a pair of narrow slits, or diffracts around a slip of card, each wavelength of light will interfere only with itself, and the spots of constructive interference will occur at different spots. However, since the spectrum is continuous, the angles \( \theta \) at which constructive interference occurs will also be continuous, and so instead of seeing definite areas of constructive or destructive interference the pattern will be blurred. The dark areas of destructive interference for one color of light will be filled in by bright areas of constructive interference for other colors of light.

The center spot is white because the path-length difference for all wavelengths is zero. All wavelengths converge here.

**Figure 20 White-light Interference Pattern**

The laser is composed of only one wavelength of light, thus the bright areas of constructive interference and the dark areas of destructive interference are clear and distinct from one another.

**Figure 21 Laser-light Interference Pattern**
The second modification to Young’s original experiment is to use a blackened glass slide with two slits etched into it, rather than the slip of card. This is more easily maneuverable; and with today’s technology, is more easily constructed and reproduced to a high precision. However, it introduces a layer of complexity that is not seen in the “pond ripple” example above, or to such a degree with Young’s setup.

Before the middle of the seventeenth century, light had been seen, and documented, only to cast sharp shadows behind obstructions. There is some evidence that Leonardo Da Vinci (1452 – 1519) is the first to reference the bending of light around corners, but it is Francesco Grimaldi in 1665 who first documents the phenomenon, names it “diffraction”, and seems to understand its implications of a wave nature of light. In Grimaldi’s experiment, a small beam of sunlight is allowed to enter a darkened room. The light is partially blocked by a small rod, and the shadow of the rod is projected on a white screen. What Grimaldi found is that the shadow was larger than what is predicted by the geometry of the system, and that there were colored fringes at the edges of the shadow. In 1665 he published his results:

*When the light is incident on a smooth white surface it will show an illuminated base IK notably greater than the rays would make which are transmitted in straight lines through the two holes. This is proved as often as the experiment is tried by observing how great the base IK is in fact and deducing by calculation how great the base NO ought to be which is formed by the direct rays. Further it should not be omitted that the illuminated base IK appears in the middle suffused with pure light, and at either extremity its light is colored.*

**Figure 22 Grimaldi’s Experiment and Results**

This bending of light around corners, or *diffraction*, is some of the first direct evidence of the wave-nature of light.

Christiaan Huygens, a late-seventeenth century Dutchman, is one of the first to be remembered for postulating a wave theory of light. Robert Hooke made a similar proposal 20 years before Huygens, which no one remembers; but then Huygens was the first to build a pendulum clock, for which Galileo gets the credit.

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2 James C. Wyant;  
[http://www.optics.arizona.edu/jcwyant/Optics505(2000)/ChapterNotes/Chapter01/chapter1.pdf](http://www.optics.arizona.edu/jcwyant/Optics505(2000)/ChapterNotes/Chapter01/chapter1.pdf)  
Diffraction is seen in many instances. The simplest is around the straight edge of an opaque object:

![Figure 23](image)

**Figure 23**

Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object. Serway and Jewitt, 6th Edition, pg. 1207

More complicated objects produce more complex diffraction patterns:

![Figure 24 (a) and (b)](image)

**Figure 24 (a) and (b)**

Figure (a) shows the diffraction of light around a razor blade. Figure (b) shows two images of the shadow of a screw in laser light. The fringes are produced by destructive interference of the diffracted light. The image on the right side is a longer exposure showing fringes within the shadow, produced by constructive and destructive interference. Hewitt, 8th Edition, pg. 521

In the early nineteenth century, Simeon Poisson, a proponent of the light-as-particle theory, said that if light truly is a wave, then a circular object illuminated by a point source of light should, by the wave theory, have a bright spot of light at the center of its shadow. Dominique Arago, only a few years later, observed this spot, thus reinforcing the wave theory of light.
Why do we see a diffraction pattern when there is only one source of light?  Huygens’ Principle proposes that every point on a wave crest – or wave front – behaves as though it itself were a point source sending out another set of wave fronts.  Thus, each wave front is made up of tinier wave fronts, or wavelets.

Thus, each wave front (whether circular or linear) consists of hundreds of “sources”.  Each of these sources sends out circular waves (whether the overall pattern is circular or linear), and a tangent line to these “wavelets” forms the new wave front...what we would usually think of as the primary wavefront moving forward in space.

Because each point on a wavefront can be thought of as a source, when a linear wave passes through an opening in a barrier (or a single slit), the wavelets from one “point source” can interfere with the wavelets from a second “point source”, similar to what we discussed earlier in the case of two-slit interference.  However, instead of having only two point sources, the wavelet theory proposes an infinite number of point sources.
Let us look at “sources” 1 and 3. The rays of light traveling from these sources at some angle $q$ will meet at a point on a screen that is a sufficiently far distance $L$ away. Ray 1 must travel a distance $a/2 \sin \theta$ further than Ray 3. If this distance, $a/2 \sin \theta$ is equal to one-half wavelength, then the waves will interfere destructively. This will be true for any two rays separated by a distance of half the slit width, $a/2$. Thus, we will find destructive interference when:

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2}$$

or

$$\sin \theta = \frac{\lambda}{a}$$

We can see that all the light rays in the top half of the opening interfere destructively all the light rays in the bottom half of the opening at an angle $\theta$ such that $\sin \theta = \frac{\lambda}{a}$. If we imagine dividing the opening into four equal areas, then the top quarter will interfere the second quarter when:

$$\frac{a}{4} \sin \theta = \frac{\lambda}{2}$$

or

$$\sin \theta = \frac{2\lambda}{a}$$

If we divided the opening into six equal parts, we would find that destructive interference occurs when:

$$\sin \theta = \frac{3\lambda}{a}$$

and extending the supposition we may conclude that destructive interference (dark spots) occurs when

$$m\lambda = a \sin \theta \quad \text{where} \quad m = 1, 2, 3, \ldots$$ \hspace{1cm} Eq. 2

Now, it is historically entrenched terminology that we refer to the interference pattern generated by a single slit as a “diffraction pattern”. This is a misnomer – diffraction is the bending of light waves around corners. The appearance of a pattern of light and dark areas behind a single slit is, as explained above, due to the interference of a light wave with itself. However, perhaps to differentiate between the one set-up and the other, we refer to “double-slit interference” and “single-slit diffraction”.

This is where the modification to Young’s original experiment shows its complications. Because a set of double slits is made up of, well, two single slits, the pattern you will see is a superposition of the double-slit interference pattern on top of the single-slit diffraction pattern. It is the single-slit diffraction pattern that generates the intensity fall-off. If the ratio of the slit-separation to slit-width, $d/a$, is an integer, one of the interference maxima will be suppressed by a diffraction minimum. Exactly why is left as an exercise for the student.

The theory for this lab was written by Jennifer LK Whalen
Experiment And Analysis:

Part A: Double Slit Interference

1. Choose slide 9165-B from the optics kit.

2. Set up the laser at one end of the optics bench, with the slide in a holder directly in front of it. You can line up the laser beam and the slits by looking from the back (laser side) of the slide. Do Not look through the slits at the laser beam! Start with double slit (A).

3. Place a screen approximately 2 meters from the double slits. Measure this distance, $L$.

4. An interference pattern of bright laser dots (maxima) and dark spaces (minima) should appear on the screen. Measure the distance between the farthest dot on the left and the farthest dot on the right. Divide this distance by [number of dots minus 1] to determine the distance, $x$, between two sequential dots.

5. Divide the distance between dots, $x$, by the distance from the slits to the screen. This is a measure of $\tan \theta$. If $x$ is quite small and $L$ is quite large, then $\tan \theta \approx \theta \approx \sin \theta$. Thus, we will use this measurement to be equal to $\sin \theta$.
6. From the interference equation, \( n\lambda = d \sin \theta \), where \( n \) is the maxima number (equal to 1 for your calculation), \( d \) is the distance between the slits, and \( \sin \theta \) is the number from step (5); determine \( \lambda \), the wavelength of the laser light.

7. Repeat steps 2-6 with slits (B), (C) and (D).

8. Present your data in a neat table. Determine, from your measurements, the average value of \( \lambda \), and its uncertainty.

9. Does your value of \( \lambda \) agree with the actual value of \( \lambda \) for a Helium-Neon laser, \( \lambda = 632.9 \) nm, within your experimental uncertainty (i.e. is \( \lambda_{\text{meas}} - \sigma < \lambda_{\text{actual}} < \lambda_{\text{meas}} + \sigma \), where \( \sigma \) is your uncertainty)?

**Part B: Single Slit Diffraction**

1. Replace the double slit slide with the single-slit source, 9165-A. Use slit (B) first.

2. As in Part A, place a screen approximately 2m from the slits, and measure this distance, \( L \).

3. A diffraction pattern of horizontal “bars” should appear on the screen, more widely spaced than the dots from Part A. The bright bars are the maxima, and the dark spaces are the minima. We use a similar equation for diffraction as for interference, but we count minima instead of maxima. Using a measurement procedure similar to that in Part A, determine the distance, \( x \), between subsequent minima. Remember: this distance may be measured as the length between the centers
of neighboring maxima, the length from the leading edge of one maximum to the leading edge of the next, or as drawn, the length between the centers of neighboring minima. Do not include the central maximum in your calculations, because it is larger than the other maxima and will throw-off your calculations.

![Diagram of double-slit interference and single slit diffraction](image)

**Figure 31**

4. From the interference equation, \( m\lambda = a \sin \theta \), where \( a \) is the slit width, and \( m \) is the minima-number, determine the wavelength of the laser light. You will have to use a similar trigonometric approximation as in the double-slit measurement of wavelength.

5. Repeat the measurement with slit (D).

6. Present your data in a neat table. Determine a second experimental value of \( \lambda \). Does your value agree with the actual value of \( \lambda \) for a He-Ne laser?

7. Determine which experiment, Part A or Part B gives a better experimental value of the wavelength of a He-Ne laser, and explain why.

**Part C. Double Slit Interference with Single Slit Diffraction.**

1. Replace the single slit source with the double slit source. Re-establish the interference pattern on the screen with slits (B). This time, take note that the double-slit interference pattern of small, evenly spaced dots is not constant, but fades in and out. If you stand back from the screen, the dots should blend into the bars of the single slit diffraction pattern that is superposed on the double slit pattern.

![Diagram of double-slit interference with single slit diffraction](image)

**Figure 32**

2. Find the first and second order minima for the (single slit) diffraction pattern.
3. Measure \( x \). Calculate \( a. (m\lambda = a \sin \theta) \) (As before, you will need to use trigonometric approximations!)

4. Estimate the uncertainty in \( a \).

5. Observe which interference maximum is suppressed by the 1st order diffraction minimum (i.e. which maximum from the double slit pattern is canceled out by the minimum of the single slit interference pattern).

6. Compare the order of this suppressed maximum with the ratio \( d/a \). Derive an equation that explains your findings.

7. Repeat (1-6) with double slits (A) and (C).

**D. Measuring the Intensity Pattern**

In this section you will directly measure the intensity as a function of displacement, \( x \), using the Vernier light probe and motion sensor with Logger Pro.

1. Clamp the light sensor onto a stand. The sensor will need to be level with the laser and pointed directly at it. Tape a piece of cardstock to the side of the rod above the light sensor so that the motion detector can pick it up. Tape a meter stick to the table, making sure that it is perpendicular to the axis of the optical bench. This will help to keep the sensor at the same distance from the source.

2. Plug in the LabPro and connect it to the computer with a USB cable. Connect the light sensor and motion sensor to the Lab Pro.

3. Open the Experiments folder on the desktop and open the file diffract.xmbl (or .cmbl). This will start the program Logger Pro3.5 and bring up the appropriate data file.

4. Set the intensity switch on the light sensor to 600 lux.

5. Place the double slit in the path of the laser beam, and project the interference pattern on a screen. Move the slide around on the carrier until you get the brightest interference pattern.

6. Remove the screen. You are ready to take data now. Two problems can arise:
   
   a. *The motion detector does not read distance correctly.* If this occurs, make sure that the cardstock is at least 40cm away from the motion detector throughout the experiment. Also make sure the detector is level with the cardstock and that the path from the detector to the cardstock is exactly perpendicular.

   b. *The light detector isn’t picking anything up.* In this case, make sure the sensor is set to 600 lux. Place the detector in the bright central maximum region and make sure that the beam is centered on the detector. Also check to make sure that the diffraction pattern is exactly horizontal. If it isn’t, then moving the
detector side to side can result in losing the signal as the pattern migrates up or down.

7. Hit the **Collect** button and slowly move the detector across the diffraction pattern, using the meter stick on the table to ensure that you do not move the detector closer or farther from the source.

8. Plot *intensity vs. displacement* for your interference pattern and turn in this plot with your report. Be sure to identify the important features of your plot (i.e. maxima and minima).

9. Replace the double slit with a single slit and repeat Steps 5-12.

10. For the single slit pattern, compare the intensity of the central maximum to the intensity of the next two largest maximums from your experimental data. Is this ratio in agreement with the value predicted for the intensity of a single slit interference pattern?

11. **Optional**: Export your double slit data to graphical analysis. Create a calculated column converting your displacement data to angle $\theta$. The theoretical formula for the diffraction pattern intensity is

$$ I(\theta) = 4I_0 \left( \frac{\sin^2 \beta}{\beta} \right) \cos^2 \alpha $$

where

$$ \beta = \frac{kb}{2} \sin \theta $$

and

$$ \alpha = \frac{ka}{2} \sin \theta $$

$k$ is the wave number, $k = \frac{2\pi}{\lambda}$.

Plot intensity vs. $\theta$, and perform a curve fit to the data using your values for $a$ and $b$ while leaving $I_0$ and $k$ as parameters. How does the value of $k$ obtained by the curve fit compare to the actual value?