Relationships Between Frequency, Capacitance, Inductance and Reactance.

Purpose:
To experimentally verify the relationships between $f$, $C$ and $X_C$. The data collected will lead to the conclusion that $X_C$ is inversely proportional to $C$ and $f$. This activity will also verify the capacitive reactance formula.

Equipment:
- Patch Cords, Alligator Clips
- 2 DMMs
- Capacitance Box
- Function Generator
- Decade Resistance Box
- Toroidal inductors (5H)
- LCR meter

Theory:
A battery connected in series with a resistor forms a simple circuit where Ohm's law can be applied. Using Ohms law, the current in the circuit is given by

$$I = \frac{\varepsilon_o}{R} \quad \text{Eq. 1}$$

Where $\varepsilon_o$ is the battery voltage, and $R$ is the resistance.

If a capacitor is placed a series circuit with a dc voltage source such as a battery, the current though the circuit will be exactly zero. If you replace the battery with an ac (sinusoidal) voltage source, then the current will not be zero. Instead it will be a sinusoidal function with amplitude given by:

$$I_o = \frac{\varepsilon_o}{X_C}, \quad \text{where} \quad X_C = \frac{1}{2\pi fC} \quad \text{Eq. 2}$$

In some ways, then the capacitor is a type of "AC resistor" with effective resistance given by $X_C$.

Proof:
The voltage drop across a capacitor is given by

$$V_C = \frac{q}{C}.$$  

Since $q = \int idt$ and $i$ is a sinusoidal of the form:

$$i(t) = I_o \sin(2\pi ft)$$

then
\[ q(t) = \int I_o \sin(2\pi ft) \, dt = \frac{I_o}{2\pi f} \cos(2\pi ft) \]

thus \( V_C \) can be written as:

\[ V_C = \frac{I_o}{2\pi fC} \cos(2\pi ft) = V_{C_{\max}} \cos(2\pi ft). \]

We define the reactance of the capacitor as:

\[ X_C = \frac{V_{C_{\max}}}{I_{\max}} \quad \text{Eq. 3} \]

Thus,

\[ X_C = \left( \frac{I_o}{2\pi fC} \right) / I_o = \frac{1}{2\pi fC} \quad \text{Eq. 4} \]

this implies that we can calculate \( X_C \) by Eq. 3.

If an inductor is placed in a series circuit with a sinusoidal voltage supply, then the current is given by:

\[ I_o = \frac{E_o}{X_L} \quad \text{where} \quad X_L = 2\pi fL \]

In the same way, an inductor can be considered an AC resistor, with effective resistance \( X_L \).

**Proof:**

The voltage drop across an inductor is given by

\[ V_L = L \frac{di}{dt}. \]

If

\[ i(t) = I_o \sin(2\pi ft), \]

then

\[ V_L = 2\pi fLI_o \cos(2\pi ft) \]

and we can define the inductive resistance \( X_L \) by:

\[ X_L = \frac{V_{I_{\max}}}{I_{\max}} \quad \text{Eq. 5} \]

thus,

\[ X_L = \frac{2\pi fLI_C}{I_o} = 2\pi fL \quad \text{Eq. 6} \]

This implies that we can calculate \( X_C \) using Eq. 5 by measuring the voltage drop across the inductor and the current through the circuit. We can also calculate \( X_L \) from Eq. 6 with knowledge of \( f \) and \( L \).
Experiment and Analysis:
Part A
1. a. Construct the circuit shown in Figure 1 below using a 1-µF capacitor for C₁. Use the LCR meter to determine a "good" value for the capacitance. Next, set the generator frequency to approximately 100 Hz. Adjust the output control for approximately 1-3 V across the capacitor. Record the current flowing through the capacitor.

Note: you may have difficulty using the DMM meter to record the current! It is likely to be rather small. An alternative strategy is to set up the circuit as shown in Figure 2. The resistance should be on the order of \( \frac{1}{2\pi fC} \). You can then measure the voltage across the resistance box and divide by the resistance (\( I = \frac{V}{R} \)) to determine the current. Keep in mind that you must use figure (1) to measure the voltage across the capacitance, and figure (1b) to measure the voltage across the resistor. Also remember not to change the resistance when you measure voltage across the capacitor.

Note that the DMM measures RMS values for current: \( I_{rms} = \frac{I_o}{\sqrt{2}} \)
(Essentially, this means that that \( I_o \) = the DMM reading multiplied by \( \sqrt{2} \).)
DMM AC Voltage readings are also rms values, so that \( V_{rms} = \frac{V_o}{\sqrt{2}} \).

Figure 1  Measuring the Voltage with the DMM
b. Using your measured current and voltage, compute the reactance of the capacitor using \( X_C = \frac{V_C}{I_C} \). You can use the DMM readings for \( V_C \) and \( I_C \) since the factor \( \sqrt{2} \) cancels out.

c. Calculate the reactance using the reactance formula

\[
X_C = \frac{1}{2\pi f C}
\]

d. Are the values computed in Steps 1b and 1c within 10 percent of each other? If not, use the LCR meter to get good values for the capacitance, and resistance. If they check out, then use the oscilloscope to determine the period and frequency of the function generator. Also be sure that you have a "good" sine wave. When you are satisfied, enter your results in Table 1.

e. Do your results confirm the reactance formula (within the tolerances of the meter, the generator and the capacitor)? That is, does \( I_C X_C = V_C \)?

2. a. Suppose the generator frequency is changed to 50 Hz while the voltage remains at 3 V. What will happen to the reactance?

b. What will happen to the circuit current?

c. Change the generator frequency to 50 Hz and record the new current in Table 1.

d. Does your answer to Step 2c support your answer to Step 2b?

e. How much current would you expect if the frequency were changed to 200 Hz and \( V_C \) remained at 3 V?

f. Change the frequency to 200 Hz and adjust the generator for 3 V across the capacitor. Record the current.
3. Change the capacitor in Fig. 1 to 0.15 µF. Repeat Steps 1 and 2. Enter your results in Table I.

4. Do your values for \( V_C \) and \( I_C X_C \) agree in each case? If not, is the agreement better at high or low frequencies? Why do you think this is?

**Part B**

1. Repeat the procedure from Part 1 using the 5H inductor in place of the capacitor. Use the LCR meter to get a "good" value for inductance. Complete Table 2 using Eqs. 5 and 6. Your values for \( X_L \) should agree within 10 or 15% (due to uncertainties in measuring AC current accurately). If you are using the resistance box to determine current, you will have to change the setting so that enough voltage is dropped across it to be measured. In general, set \( R_{box} \) approximately equal to \( 2 \pi f L \) for whatever frequency you are using.

![Diagram of electric circuit](image)

**Figure 3**

2. Check to see if the reactance determined above agrees with the value calculated by the reactance formula. Calculate and record the reactance using the formula \( X_L = 2 \pi f L \). The reactance calculated here should be within 15% of the reactance you listed in Table 2. If it is not, recheck your measurements and your calculations.

3. Change the frequency of the generator to 500 Hz. Measure the output voltage of the generator. Note that the output voltage is likely to drop. Make the measurements and calculation necessary to complete the second row of the table. Does your data table support your predictions about what happens to the reactance and the current?
4. Change the generator frequency to 1000 Hz. From the reactance you determined in the first two rows of Table 2, predict and record the reactance you will have at 1000 Hz. Complete the third row of the table to check your prediction.

Results:
Write at least one paragraph describing the following:
- what you expected to learn about the lab (i.e. what was the reason for conducting the experiment?)
- your results, and what you learned from them
- Think of at least one other experiment might you perform to verify these results
- Think of at least one new question or problem that could be answered with the physics you have learned in this laboratory, or be extrapolated from the ideas in this laboratory.
Clean-Up:
Before you can leave the classroom, you must clean up your equipment, and have your instructor sign below. How you divide clean-up duties between lab members is up to you.

Clean-up involves:
- Completely dismantling the experimental setup
- Removing tape from anything you put tape on
- Drying-off any wet equipment
- Putting away equipment in proper boxes (if applicable)
- Returning equipment to proper cabinets, or to the cart at the front of the room
- Throwing away pieces of string, paper, and other detritus (i.e. your water bottles)
- Shutting down the computer
- Anything else that needs to be done to return the room to its pristine, pre lab form.

I certify that the equipment used by ________________________ has been cleaned up.

______________________________ , _______________.

______________________________ , __________________.

(student’s name) (instructor’s name) (date)
### LPC Physics

**Relationships between $f$, $C$, $L$ and $X$.**

**TABLE 1**

<table>
<thead>
<tr>
<th>C</th>
<th>$I_C$</th>
<th>$V_C$</th>
<th>$f$</th>
<th>$X_C$(theory)</th>
<th>$X_C$(exp)</th>
<th>%diff</th>
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**TABLE 2**

<table>
<thead>
<tr>
<th>Frequency $f$</th>
<th>Current $I$</th>
<th>Voltage across $L_1$, $V_{L_1}$ (measured)</th>
<th>Inductive Reactance $X_L = 2\pi f L$</th>
<th>Inductive reactance $X_L = \frac{V_L}{I}$</th>
<th>% difference</th>
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